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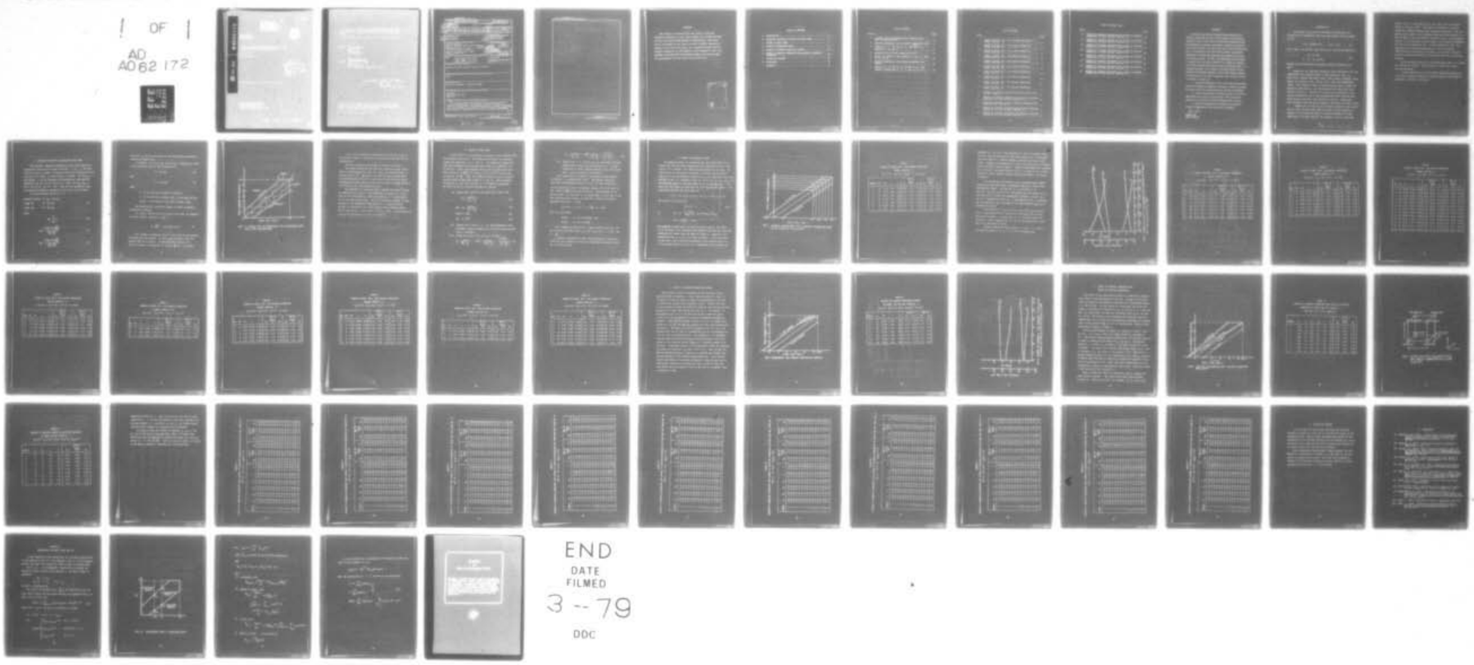

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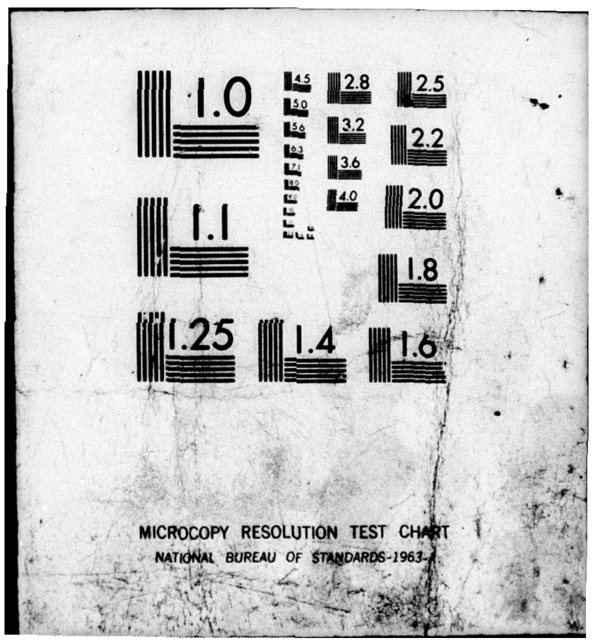
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This report documents a new analytical procedure which can be used to analyze and evaluate sequential probability ratio test plans used for reliability demonstration. The methodology developed is capable of evaluating such test plans when the two lines which define the accept, reject regions are either parallel or nonparallel, with or without truncation considerations.

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ABSTRACT

This report is concerned with the study of truncated sequential probability ratio tests (TSPRT) for reliability testing when the failure distribution is exponential. The study uses a new method for the exact analysis of TSPRT. Effects of changes in the truncation point, the truncation region and the decision boundaries on the producer's and consumer's risks, the expected test time (ETT) and expected number of failures (ENF) are investigated for test plans from MIL-STD-781B.

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EVALUATION

In-the-house investigations indicated that the formulation of sequential probability ratio tests (SPRTs) based on the structure of nonparallel accept, reject lines with truncation considerations could have advantages over the more conventionally structured parallel line form of SPRT (the SPRT is the most widely used reliability demonstration test). The objective of this effort was to develop a quantitative methodology to evaluate test plans based upon nonparallel structures. This objective was met. Not only is the methodology developed capable of evaluating this case, but in addition, it is capable of providing for a direct analytical evaluation of the more conventional SPRTs. The procedure developed, in fact, appears to be more efficient and cost-effective than procedures presently in use for this purpose. Besides the dissemination of the report to potential users, follow-on activity is currently in progress to:

a. Include the methodology as part of the "RADC Compu-Standards Program" (a computerized compendium of procedures intended to implement and support reliability and maintainability standards and handbooks).

b. Utilize the methodology to develop new types of reliability demonstration tests (based on the nonparallel SPRT structure) for proposed use in MIL-STD-781 (Reliability Demonstration) revisions.

c. Utilize the methodology in-the-house to custom design reliability demonstration tests for specific procurements.

Jerome Klion

JEROME KLION
Project Engineer

1. INTRODUCTION

We consider the problem of reliability testing when the failure time is exponential with mean time between failures (MTBF) θ, viz.

$$f(t|\theta) = \frac{1}{\theta}\exp(-t/\theta), \qquad t \geq 0, \ \theta > 0. \tag{1}$$

Such a test traditionally takes the form of testing the hypothesis

$$\begin{aligned} H_0 &: \theta = \theta_0 \\ \text{vs} \quad H_1 &: \theta = \theta_1 < \theta_0 \end{aligned} \tag{2}$$

because of the one-to-one relationship between reliability and MTBF.

Sequential and truncated sequential tests (see below) for this problem have been studied widely in the literature; see, e.g. Wald (1947), Epstein and Sobel (1954), Weiss (1953), Anderson (1960), Woodal and Kurkjian (1962), Aroian (1963, 1976) and Raghavachari (1965). The purpose of these investigations has been to study the Operating Characteristic (OC) and Expected Test Time (ETT) functions for the test. Because of the practical advantages of truncating a sequential test at some prespecified point, MIL-STD-781B gives several such plans for reliability testing. These plans were developed using the work of Epstein, Patterson and Qualls (1963).

Because of the lack of exact results, studies in the literature on such truncated tests have not dealt with the effects of changes in the truncation region or decision boundaries on the quantities of interest such as the producer's risk (α), the con-

sumer's risk (β), the expected test time (ETT) and the expected number of failures (ENF). This type of information is usually needed for trade-off studies when selecting a test plan. The purpose of this report is to investigate how and to what extent α, β, ETT and ENF are affected when the truncation point, the truncation region, or the intercepts and the slopes of the decision boundaries are varied in a meaningful way. In this study we use some newly developed exact results (see Appendix A) for this purpose. These results are analytically tractable, and provide a computationally economical procedure for conducting the above sensitivity analyses.

We use primarily Test No. 3 of MIL-STD-781B (1967) as a basis for sensitivity studies. Some analyses for other test plans are also presented.

A brief description of the truncated sequential probability ratio test is given in Section 2 and the results of the study are presented in Sections 3 through 6.

2. TRUNCATED SEQUENTIAL PROBABILITY RATIO TEST

The truncated sequential probability ratio test consists of sequentially observing the times to failure, $t_1, t_2, \ldots,$ and plotting the cumulative number of failures versus the cumulative failure time until the accept or reject region is reached. Let $(\bar{A}_1, \bar{A}_2, \ldots)$ and $(\bar{R}_1, \bar{R}_2, \ldots)$ be two sequences of predetermined, non-negative constants such that $\bar{A}_i \leq \bar{R}_i$ for all i. Also, let $\bar{A}_{N*} = \bar{R}_{N*}$ at $N* < \infty$ and $\bar{A}_i < \bar{R}_i$ for all $i < N*$. Then for the exponential case the truncated sequential probability ratio test is as follows.

At the Nth failure $(N = 0, 1, 2, \ldots)$,

$$\begin{aligned} &\text{continue testing} \quad &&\text{if} \quad R_N < W_N < A_N, \\ &\text{accept } H_0 &&\text{if} \quad W_N \geq A_N, \\ &\text{reject } H_0 &&\text{if} \quad W_N \leq R_N, \end{aligned} \tag{3}$$

where

$$W_N = \sum_{i=1}^{N} t_i \tag{4}$$

$$A_N = \frac{\ln \bar{A}_N - N \ln\left(\frac{\Theta_0}{\Theta_1}\right)}{\left(\frac{1}{\Theta_0} - \frac{1}{\Theta_1}\right)} \tag{5}$$

$$R_N = \frac{\ln \bar{R}_N - N \ln\left(\frac{\Theta_0}{\Theta_1}\right)}{\left(\frac{1}{\Theta_0} - \frac{1}{\Theta_1}\right)}. \tag{6}$$

Note that A_N and R_N are points on the acceptance and rejection boundaries respectively.

In general, the accept and reject lines, respectively, prior to the truncation region, can be written as

$$T = h_2 + S_2 N \tag{7}$$

and

$$T = -h_1 + S_1 N \tag{8}$$

where

N is the cumulative number of failures,

T is the cumulative failure time (in multiples of θ_0),

$S_1(S_2)$ is the slope of the reject (accept) line.

For obtaining the truncation region, as shown in Figure 1, we proceed as follows:

(i) Draw a line OL from the origin 0 such that its equation is $T = SN$, where, letting $d = \theta_0/\theta_1$,

$$S = \frac{\ell n d}{d-1} \quad \text{(in units of } \theta_0\text{)} \tag{9}$$

(ii) Choose a truncation time T^* which will be the maximum allowable time for testing. In this study we require that the quantity $\frac{T^*}{S}$ be an integer. If the specified values do not yield an integer, we round off T^* to make $\frac{T^*}{S} = N^*$, an integer.

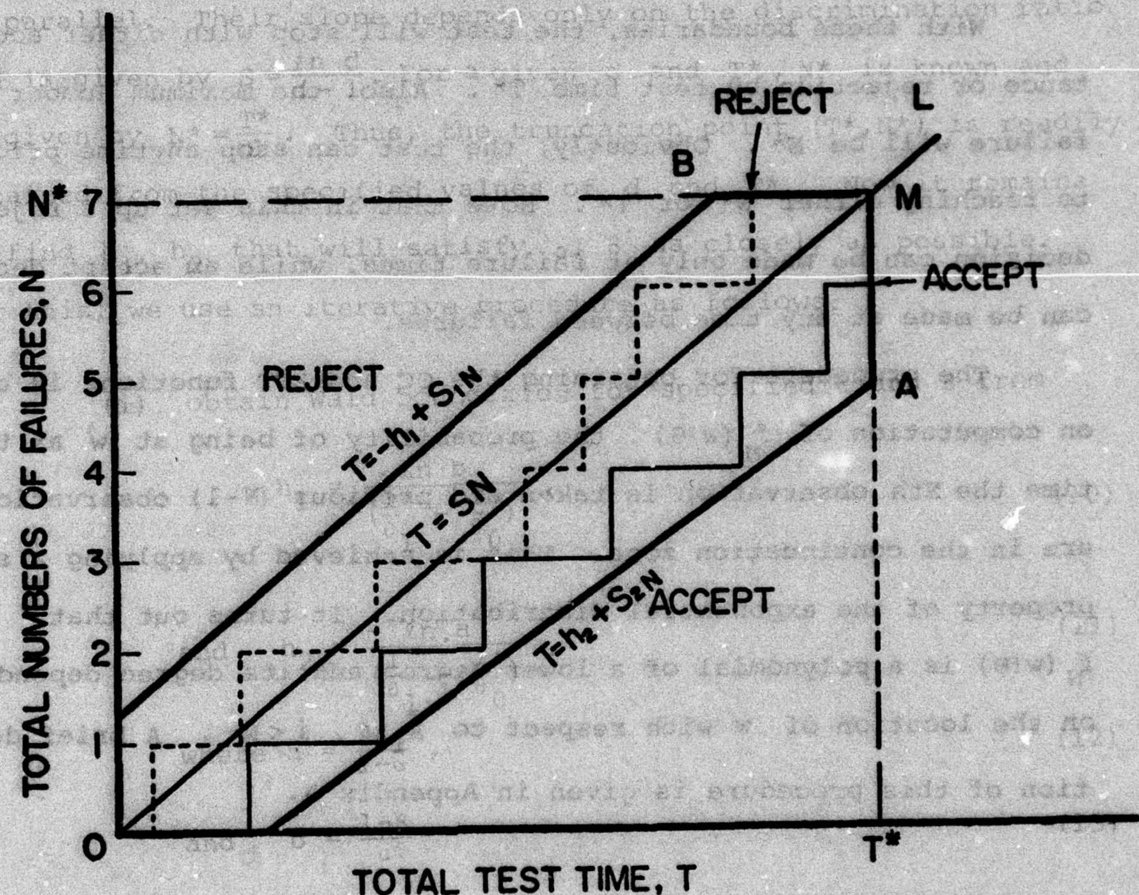

FIG. 1 A TYPICAL SET OF BOUNDARIES FOR TRUNCATED SPRT WITH TWO REALIZATIONS

(iii) Draw a horizontal line from $N = N^*$ and let B and M be the points where it intersects the reject line and the line OL, respectively.

Then the truncation region BMA is as shown in Figure 1.

With these boundaries, the test will stop with either acceptance or rejection by test time T^*. Also, the maximum number of failure will be N^*. Obviously, the test can stop anytime prior to reaching either N^* or T^*. Note that in this set up a reject decision can be made only at failure times, while an accept decision can be made at any time between failures.

The procedure for obtaining the OC and ETT functions is based on computation of $f_N(w|\theta)$, the probability of being at w at the time the Nth observation is taken and previous (N-1) observations are in the continuation zone. This is achieved by applying a simple property of the exponential distribution. It turns out that $f_N(w|\theta)$ is a polynomial of a lower degree and its degree depends on the location of w with respect to $\overline{A}_i$'s, $i < N^*$. A brief description of this procedure is given in Appendix A.

3. DESIGN OF TEST PLANS

In this section we investigate the design of the truncated SPRT as described in Section 2. The purpose is to derive the decision regions for specified α, β, d and T*. For purposes of design we take the acceptance and rejection lines, prior to truncation, to be parallel. Their slope depends only on the discrimination ratio and is given by $S = \frac{\ln d}{d-1}$. For a given d and T*, N* is known and is given by $N^* = \frac{T^*}{S}$. Thus, the truncation point (T*,N*) is readily obtained from the specified values of d and T*. Now it remains to find h_1, h_2 that will satisfy α, β as closely as possible. For this, we use an iterative procedure as follows:

(i) obtain Wald boundaries for specified α and β from

$$h_1 = \frac{\ln b}{\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)}, \tag{10}$$

and $$h_2 = \frac{-\ln a}{\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)}, \tag{11}$$

where $$a = \frac{\beta}{1-\alpha}, \tag{12}$$

and $$b = \frac{1-\beta}{\alpha} \tag{13}$$

(ii) Compute exact risks α', β' for these boundaries with truncation region determined by T* and N* using equation (A-2) of Appendix A.

(iii) Choose new values of h_1 and h_2 as follows:

$$h_1 = \frac{\ln b}{\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)} + \frac{\alpha - \alpha'}{\alpha} \cdot \left[\frac{\ln b}{\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)} - \frac{\ln\left(\frac{1-\beta'}{\alpha'}\right)}{\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)}\right], \tag{14}$$

$$h_2 = \frac{-\ln a}{(\frac{1}{\theta_1} - \frac{1}{\theta_0})} + \frac{\beta-\beta'}{\beta}\left[\frac{-\ln a}{(\frac{1}{\theta_1} - \frac{1}{\theta_0})} - \frac{-\ln(\frac{\beta'}{1-\alpha'})}{(\frac{1}{\theta_1} - \frac{1}{\theta_0})}\right] \tag{15}$$

(iv) compute new α', β' for the h_1, h_2 from step (iii) and compare these values with the desired α, β. If the difference is within desired accuracy, say, .01, we are done. If not, go back to step (iii).

For illustration, we consider the design of plans similar to Test No. 3 in MIL-STD-781B (1967). The desired values of risks and discrimination ratio are $\alpha = \beta = .10$ and $d = \theta_0/\theta_1 = 2$.

Let the specified value of truncation time be $T^* = 11$ (in units of θ_0). Then $S = \frac{\ln 2}{2-1} = .693$ and $N^* = 15.87$. Since we want N^* to be an integer, we round off N^* to 16 and take $T^* = 16 \times .693 = 11.09$. The truncation point becomes (11.09, 16). Using the iterative procedure given above, we get

$$h_2 = 2.55, \quad h_1 = 2.37, \quad \alpha' = .098, \quad \beta' = .104$$

and the lines become

Accept: $T = -2.55 + 0.693N$, and

Reject: $T = 2.37 + 0.693N$.

Now, suppose we take $N^* = 15$. Then $T^* = 15 \times .693 = 10.4$ and the iterative procedure gives $h_2 = 2.55$, $h_1 = 3.03$, $\alpha' = .100$, and $\beta' = .100$.

It should be pointed out that these boundaries, obtained by using our exact method, are different from the boundaries of Test No. 3 in MIL-STD-781B.

4. EFFECT OF TRUNCATION POINT

An important reason for truncating the usual Wald SPRT is to insure that the test does terminate by the truncation time T*. The choice of T*, however, will effect the performance of the test. In this section we investigate the effect of varying the truncation time T* and the corresponding values of N* on the following quantities: producer's risk (α), consumer's risk (β), ETT at θ_0, θ_1, and $(\theta_0+\theta_1)/2$, the expected number of failures (ENF) at θ_0, θ_1 and $(\theta_0+\theta_1)/2$. As a basis for this study, we take Test No. 3 of MIL-STD-781B (1967). The specified risks etc. for this test are

$$\alpha = \beta = .10,\ \theta_0/\theta_1 = 2,\ N^* = 16,\ h_1 = 1.75,\ h_2 = 2.20$$

The value of T* is given by

$$T^* = N^* \cdot S \tag{16}$$

or

$$T^* = N^* \cdot \frac{\ln\ \theta_0/\theta_1}{\theta_0/\theta_1 - 1} \quad \text{(in multiples of } \theta_0\text{)}$$

$$T^* = 16\left(\frac{\ln 2}{2-1}\right) = 11.09 \ .$$

For purposes of this study, we vary N* from 13 to 19. The values of T* for each N* are calculated from Equation (16) and the resulting boundaries for these cases are shown in Figure 2. Results of the computations for α, β, etc. are given in Table 1. From this table we note that α and β monotically decrease as N* changes from 13 to 19 (T* changes from 9.01 to 13.17), α decreases by 12.9% while the

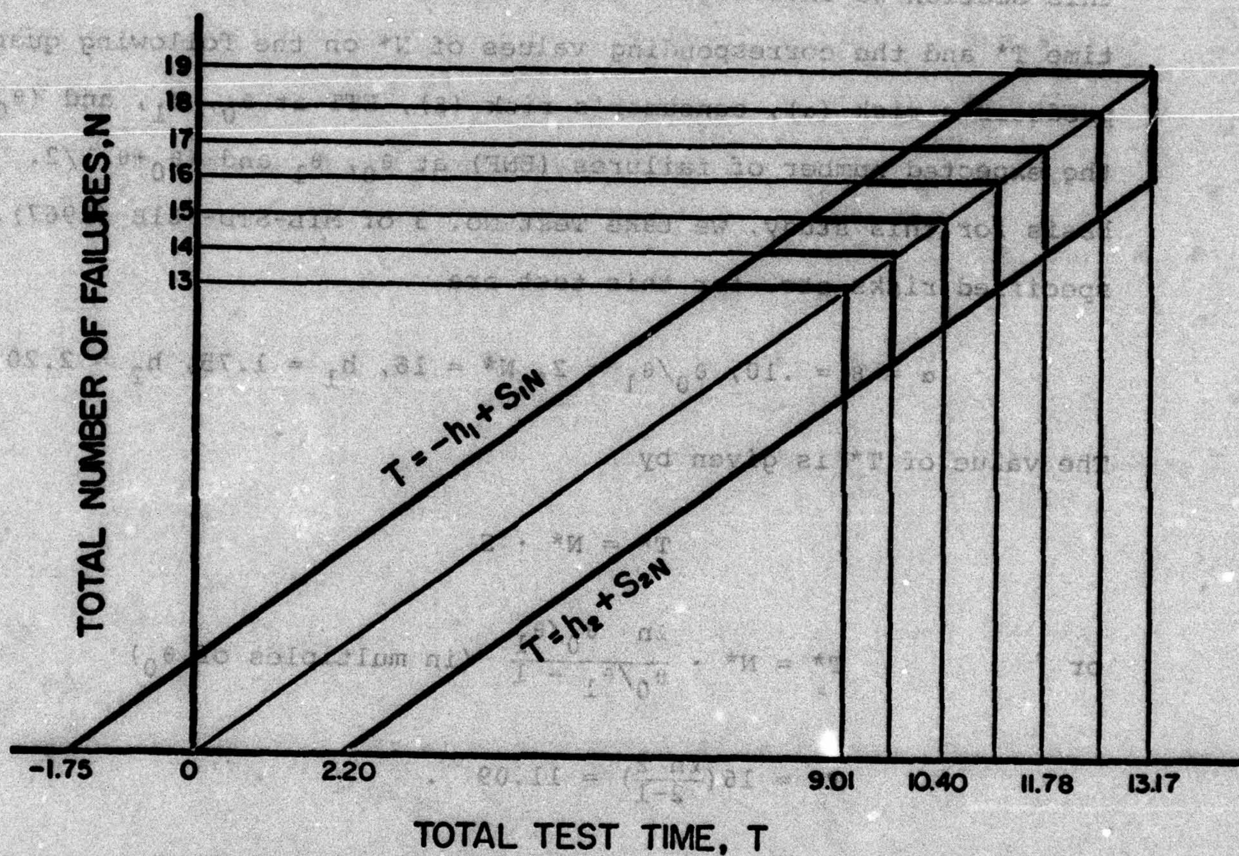

FIG. 2 DECISION BOUNDARIES WITH A VARYING TRUNCATION POINT ($S_1 = S_2 = 0.693$, $h_1 = 1.75$, $h_2 = 2.20$)

TABLE 1

VALUES OF RISKS, ETC., FOR VARIOUS TRUNCATION POINTS (TEST NO. 3)

$(h_1 = 1.75,\ h_2 = 2.2,\ \theta_0/\theta_1 = 2,\ S = 0.693)$

					ETT at			ENF at		
Number	N*	T*	α	β	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1
1	13	9.01	.154	.115	4.96	5.24	3.72	4.96	6.99	7.45
2	14	9.07	.149	.111	5.05	5.39	3.79	5.05	7.19	7.58
3	15	10.40	.144	.109	5.13	5.53	3.85	5.13	7.37	7.70
4	16	11.09	.141	.107	5.19	5.64	3.89	5.19	7.52	7.79
5	17	11.78	.138	.105	5.24	5.74	3.93	5.24	7.65	7.87
6	18	12.48	.136	.104	5.28	5.83	3.97	5.28	7.77	7.93
7	19	13.17	.134	.102	5.32	5.90	3.99	5.32	7.87	7.98

decrease in β is 11.3%. This decrease in α and β is obvious from the fact that the continuation zone gets larger as N^* is increased and hence there is a reduced chance of committing errors of both kinds. However, this reduction in α and β is at the cost of increased ETT and ENF as seen in Table 1. Both of these quantities increase by 7.2% when N^* is changed from 13 to 19. In order to get a visual picture of these changes, selected results from Table 1 are shown in Figure 3.

To see further how changes in the truncation point effect results for other test plans, we consider Tests No. 2 and 5 from MIL-STD-781B (1967). For Test No. 2, $\alpha = \beta = 0.20$, $d = 1.5$, $h_1 = 2.27$, $h_2 = 2.79$ and $N^* = 19$. The results for N^* from 16 to 22 (T^* from 12.97 to 17.84) are summarized in Table 2. We see that as N^* is changed from 16 to 22, α decreases by 8.3%, β decreases by 4.8% and ETT at θ_0 increases by 8.4%, while ETT at $(\theta_0 + \theta_1)/2$ increases by 10.7%.

For Test No. 5, $\alpha = \beta = .10$, $d = 3$, $h_1 = 0.91$, $h_2 = 1.29$ and $N^* = 7$. The results for N^* from 4 to 10 (T^* from 2.20 to 5.49) are given in Table 3. In this case, as T^* increases from 2.20 to 5.49, α and β decrease by 47.8 and 46.2% respectively, while ETT at θ_0 increases by 57.8%.

Similar results for test numbers 1, 4, 4A, 6, 7, 8 and 9 are given in Tables 4, 5, 6, 7, 8, 9 and 10 respectively.

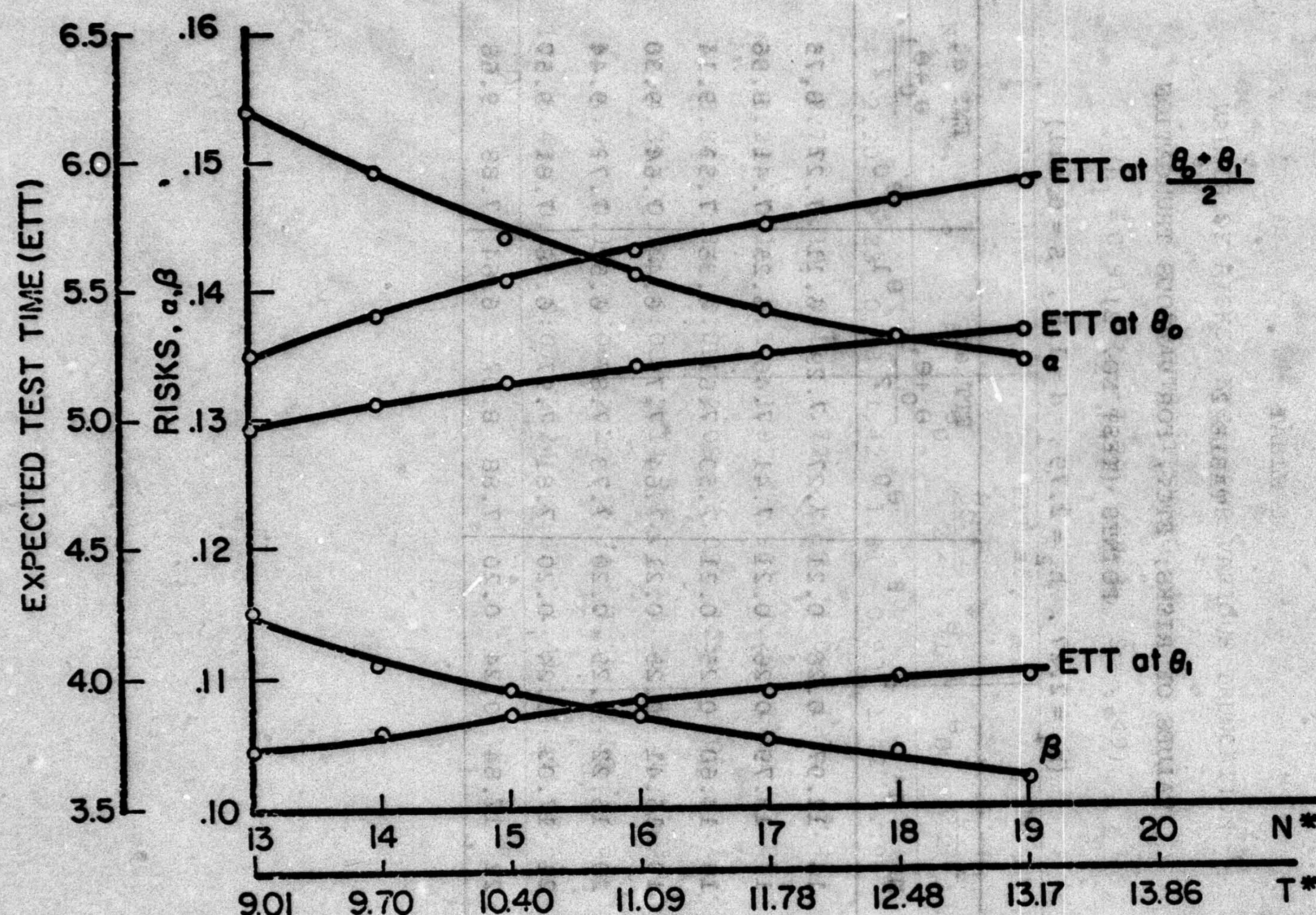

FIG. 3 CHANGE IN α, β, ETT at θ_0, θ_1 and $\frac{\theta_0+\theta_1}{2}$ AS A FUNCTION OF N^* AND T^*
($h_1 = 1.75$, $h_2 = 2.20$, $d = 2$, $S = .693$)

TABLE 2

VALUES OF RISKS, ETC., FOR VARIOUS TRUNCATION POINTS (TEST NO. 2)

($h_1 = 2.27$, $h_2 = 2.79$, $d = 1.5$, $S = 0.811$)

No.	N*	T*	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1	ENF at θ_0	ENF at $\frac{\theta_0+\theta_1}{2}$	ENF at θ_1
1	16	12.97	0.26	0.21	7.27	7.29	6.11	7.27	8.75	9.20
2	17	13.79	0.26	0.21	7.41	7.46	6.25	7.41	8.96	9.38
3	18	14.60	0.25	0.21	7.53	7.61	6.35	7.53	9.14	9.53
4	19	15.41	0.25	0.21	7.64	7.75	6.44	7.64	9.30	9.66
5	20	16.22	0.25	0.20	7.73	7.87	6.52	7.73	9.44	9.78
6	21	17.03	0.24	0.20	7.81	7.97	6.58	7.81	9.57	9.88
7	22	17.84	0.24	0.20	7.88	8.07	6.64	7.88	9.68	9.96

TABLE 3

VALUES OF RISKS, ETC., FOR VARIOUS TRUNCATION

POINTS (TEST NO. 5)

($h_1 = 0.91$, $h_2 = 1.29$, $d = 3$, $S = .549$)

No.	N*	T*	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1	ENF at θ_0	ENF at $\frac{\theta_0+\theta_1}{2}$	ENF at θ_1
1	4	2.20	0.23	0.13	1.42	1.31	0.90	1.42	1.96	2.69
2	5	2.75	0.19	0.11	1.72	1.65	1.09	1.72	2.47	3.27
3	6	3.29	0.16	0.10	1.92	1.90	1.22	1.92	2.85	3.65
4	7	3.84	0.14	0.09	2.05	2.09	1.30	2.05	3.13	3.89
5	8	4.39	0.13	0.08	2.14	2.23	1.35	2.14	3.34	4.06
6	9	4.94	0.12	0.08	2.20	2.34	1.39	2.20	3.51	4.16
7	10	5.49	0.12	0.07	2.24	2.42	1.41	2.24	3.63	4.23

TABLE 4

VALUES OF RISKS, ETC., FOR VARIOUS TRUNCATION

POINTS (TEST NO. 1)

($h_1 = 4.4$, $h_2 = 4.42$, $d = 1.5$, $\varsigma = 0.81$)

					ETT at			ENF at		
No.	N*	T*	α	β	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1
1	35	28.38	0.134	0.129	16.51	18.61	14.91	16.51	22.33	22.36
2	36	29.19	0.131	0.127	16.66	18.87	15.05	16.66	22.65	22.58
3	37	30.00	0.128	0.126	16.81	19.13	15.19	16.81	22.95	22.79
4	38	30.82	0.126	0.124	16.94	19.37	15.33	16.94	23.25	22.99
5	39	31.63	0.123	0.123	17.07	19.61	15.45	17.07	23.53	23.18
6	40	32.44	0.121	0.122	17.19	19.84	15.57	17.19	23.80	23.35
7	41	33.25	0.119	0.121	17.30	20.05	15.68	17.30	24.06	23.52
8	42	34.06	0.117	0.119	17.41	20.26	15.78	17.41	24.32	23.67
9	43	34.87	0.116	0.118	17.51	20.46	15.88	17.51	24.56	23.82
10	44	35.68	0.114	0.117	17.60	20.66	15.97	17.60	24.79	23.95
11	45	36.49	0.112	0.116	17.69	20.84	16.06	17.69	25.01	24.08
12	46	37.30	0.111	0.116	17.78	21.02	16.14	17.78	25.23	24.21
13	47	38.11	0.109	0.115	17.86	21.19	16.21	17.86	25.43	24.32

TABLE 5

VALUES OF RISKS, ETC., FOR VARIOUS TRUNCATION

POINTS (TEST NO. 4)

($h_1 = 1.4$, $h_2 = 1.04$, $d = 2.0$, $S = .693$)

					ETT at			ENF at		
No.	N*	T*	α	β	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1
1	6	4.16	0.263	0.200	2.30	2.24	1.76	2.30	2.98	3.52
2	7	4.85	0.252	0.196	2.39	2.35	1.84	2.39	3.14	3.67
3	8	5.55	0.244	0.194	2.46	2.44	1.89	2.46	3.25	3.77
4	9	6.24	0.239	0.193	2.50	2.49	1.92	2.50	3.33	3.84
5	10	6.93	0.236	0.192	2.53	2.54	1.94	2.53	3.38	3.88

TABLE 6

VALUES OF RISKS, ETC., FOR VARIOUS TRUNCATION POINTS (TEST NO. 4A)

($h_1 = 0.89$, $h_2 = 0.98$, $d = 3.0$, $S = 0.55$)

					ETT at			ENF at		
No.	N*	T*	α	β	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1
1	2	1.10	0.293	0.168	0.88	0.81	0.59	0.88	1.22	1.76
2	3	1.65	0.209	0.161	1.17	1.15	0.87	1.17	1.72	2.61
3	4	2.20	0.159	0.158	1.34	1.38	1.05	1.34	2.07	3.16

TABLE 7

VALUES OF RISKS, ETC., FOR VARIOUS TRUNCATION POINTS (TEST NO. 6)

($h_1 = 0.55$, $h_2 = 0.36$, $d = 5.0$, $S = 0.40$)

No.	N*	T*	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1	ENF at θ_0	ENF at $\frac{\theta_0+\theta_1}{2}$	ENF at θ_1
1	2	0.80	0.196	0.109	0.58	0.55	0.33	0.58	0.91	1.64
2	3	1.21	0.152	0.101	0.66	0.65	0.38	0.66	1.09	1.88
3	4	1.61	0.137	0.098	0.69	0.70	0.39	0.69	1.16	1.97
4	5	2.01	0.132	0.097	0.70	0.72	0.40	0.70	1.20	1.99
5	6	2.41	0.131	0.097	0.70	0.73	0.40	0.70	1.21	2.00

TABLE 8

VALUES OF RISKS, ETC., FOR VARIOUS TRUNCATION

POINTS (TEST NO. 7)

($h_1 = 2.10$, $h_2 = 1.62$, $d = 1.5$, $S = 0.81$)

					ETT at			ENF at		
No.	N*	T*	α	β	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1
1	3	2.43	0.438	0.295	2.01	1.88	1.69	2.01	2.26	2.54
2	4	3.24	0.407	0.287	2.61	2.47	2.21	2.61	2.96	3.31
3	5	4.05	0.382	0.279	3.09	2.94	2.62	3.09	3.53	3.93
4	6	4.87	0.364	0.273	3.47	3.32	2.94	3.47	3.99	4.42
5	7	5.68	0.349	0.268	3.77	3.63	3.20	3.77	4.36	4.81
6	8	6.49	0.337	0.264	4.02	3.89	3.41	4.02	4.66	5.11
7	9	7.30	0.328	0.261	4.21	4.09	3.57	4.21	4.91	5.36

TABLE 9

VALUES OF RISKS, ETC., FOR VARIOUS TRUNCATION

POINTS (TEST NO. 8)

($h_1 = 0.86$, $h_2 = 6.0$, $d = 2.0$, $S = 0.69$)

No.	N*	T*	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1	ENF at θ_0	ENF at $\frac{\theta_0+\theta_1}{2}$	ENF at θ_1
1	2	1.39	0.362	0.287	0.94	0.89	0.77	0.94	1.18	1.53
2	3	2.08	0.274	0.317	1.25	1.24	1.12	1.25	1.65	2.25
3	4	2.77	0.215	0.339	1.48	1.54	1.46	1.48	2.05	2.93

TABLE 10

VALUES OF RISKS, ETC., FOR VARIOUS TRUNCATION

POINTS (TEST NO. 9)

($h_1 = 2.0$, $h_2 = 3.12$, $d = 1.25$, $S = 0.893$)

					ETT at			ENF at		
No.	N*	T*	α	β	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1
1	6	5.36	0.410	0.384	3.81	3.79	3.70	3.81	4.21	4.62
2	7	6.25	0.386	0.394	4.26	4.27	4.19	4.26	4.74	5.24
3	8	7.14	0.365	0.403	4.65	4.69	4.63	4.65	5.21	5.78
4	9	8.03	0.347	0.412	4.99	5.07	5.01	4.99	5.63	6.26
5	10	8.93	0.331	0.419	5.29	5.39	5.35	5.29	5.99	6.68
6	11	9.82	0.317	0.426	5.55	5.68	5.64	5.55	6.31	7.05
7	12	10.71	0.305	0.432	5.77	5.93	5.89	5.77	6.59	7.37

5. EFFECT OF CHANGING TRUNCATION REGION

The truncation regions considered so far have been of the type described in Section 2. From a practical point of view, it is useful to know the extent to which the performance of the test is affected by changing the truncation region. Towards this end, in this section we study the effect of changing one of the truncation boundaries on risks, ETT and ENF for Test No. 3. We take $N^* = 16$ corresponding to a $T^* = 11.09$ and vary another truncation point Q from 16 to 10 to generate new rejection boundaries as shown in Figure 4. Note that T^* is kept constant at 11.09. As before, the values of α, β, ETT and ENF are computed for each case and are given in Table 11. We notice that as Q is changed from 16 to 10, α increases from .141 to .155, an increase of 9.9% and β decreases from .107 to .101, a decrease of 5.6%. The increase in α is caused by the fact that the reduction in the continuation region and an increase in the rejection region leads to an increased number of rejections. β decreases because as Q is decreased, there is a decreasing chance of accepting $\theta=\theta_0$ when in fact $\theta=\theta_1$. Also, we see that ETT and ENF both decrease, leading to an earlier and cheaper test stoppage because of the reduced continuation zone. Some selected results from Table 11 are shown in Figure 5. Thus, we see that by judiciously reducing the continuation zone, we can cut down the test effort and the consumer's risk at the cost of a somewhat higher producer's risk.

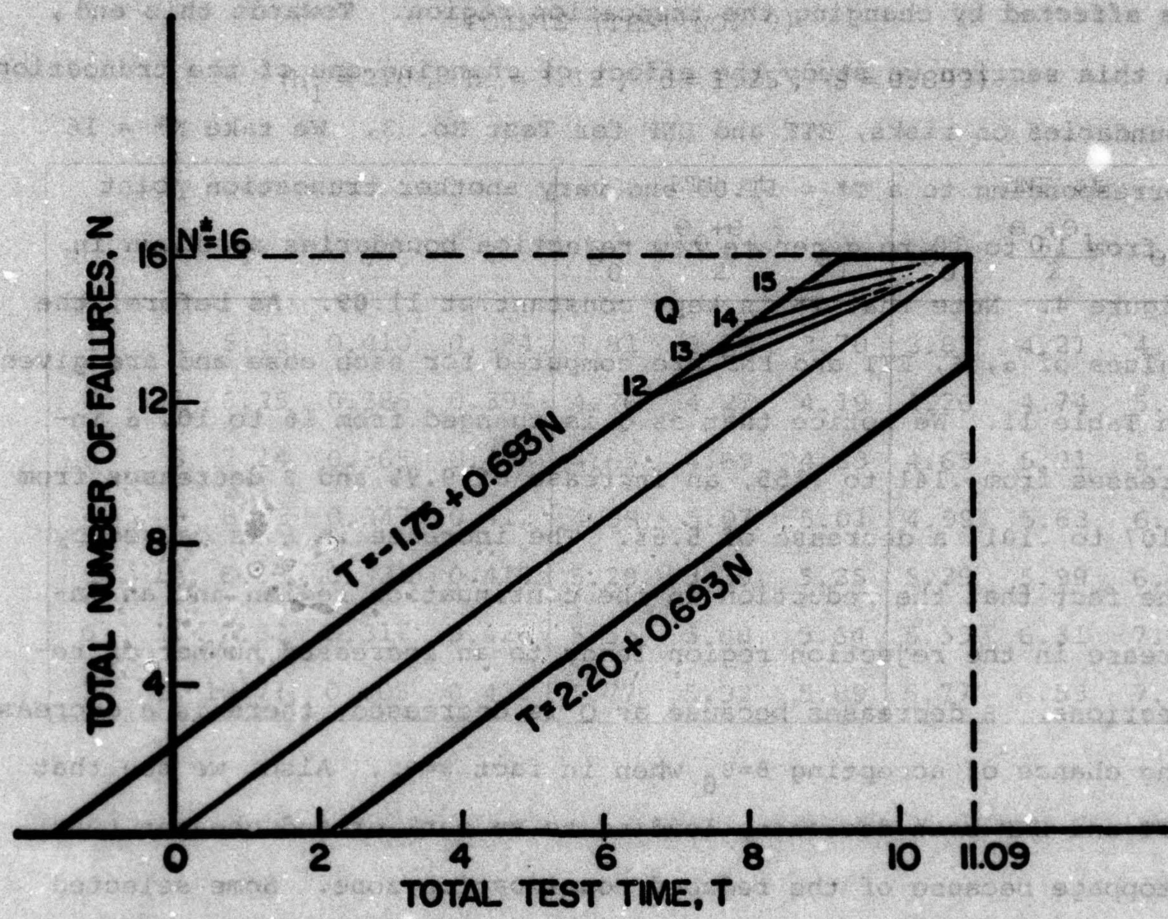

FIG. 4 BOUNDARIES FOR VARIOUS TRUNCATION POINTS, Q

TABLE 11

EFFECTS OF CHANGING TRUNCATION REGION

ON RISKS, ETT AND ENF (PLAN NO. 3)

(h_1=1.75, h_2=2.20, N^*=16, θ_0/θ_1=2, S=0.693)

Number	Q	T*	α	β	ETT at θ_0	ETT at θ_1	ENF at θ_0	ENF at θ_1
1	16	11.09	.141	.107	5.19	3.89	5.19	7.79
2	15	11.09	.141	.107	5.19	3.89	5.19	7.79
3	14	11.09	.143	.106	5.18	3.87	5.18	7.75
4	13	11.09	.145	.105	5.17	3.85	5.17	7.70
5	12	11.09	.148	.104	5.15	3.82	5.15	7.65
6	11	11.09	.152	.102	5.13	3.79	5.13	7.58
7	10	11.09	.155	.101	5.11	3.75	5.11	7.50

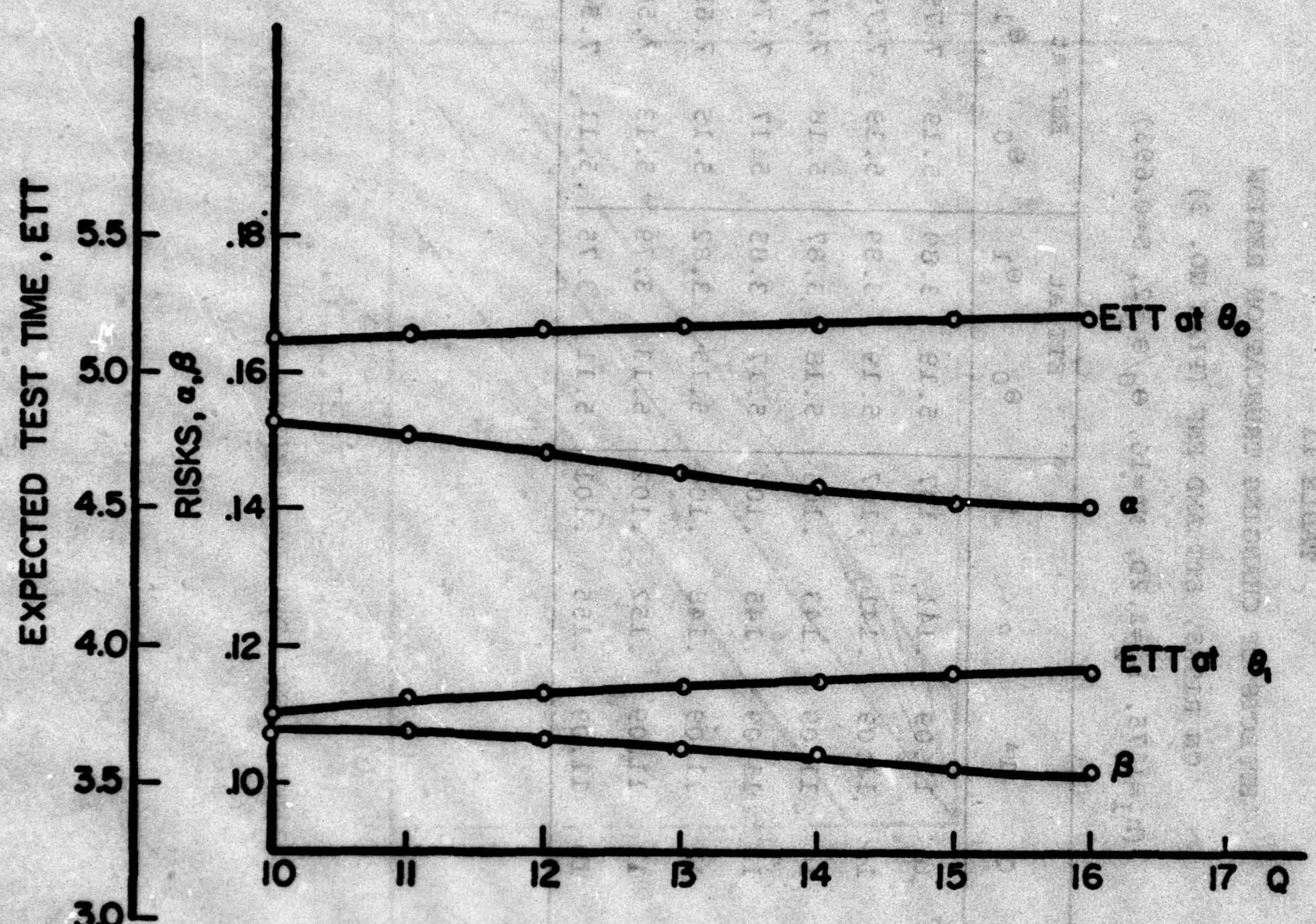

FIG. 5 EFFECT OF VARYING Q FOR CONSTANT T^* ON α, β ETT at θ_0 and θ_1 (h_1 = 1.75, h_2 = 2.20, N^* = 16, d = 2, S = .693)

6. EFFECT OF CHANGING INTERCEPTS AND SLOPES OF DECISION BOUNDARIES

The nature of the continuation region in a SPRT has a significant effect on the performance of the test. Changes in the intercepts and slopes of the decision boundaries will undoubtedly effect α, β, ETT and ENF. In this section we investigate the nature and extent of the effect of changing h_1, h_2, S_1, and S_2 on α, β, ETT and ENF. As a base point, we keep the truncation point at $N^* = 16$ and $T^* = 11.09$. The slopes of the acceptance and rejection lines are changed from .6 to .8 while the corresponding intercepts are changed from 1.65 to 1.85 and from 2.1 to 2.3, respectively. Four of these boundaries are shown in Figure 6.

Let us first consider the effect of changing h_1 ($1.65 \rightarrow 1.85$), h_2 ($2.1 \rightarrow 2.3$), and S_1 ($0.6 \rightarrow 0.8$) while keeping S_2 constant at 0.7. The results corresponding to these values are given in Table 12. In order to appreciate the effect of h_1, h_2 and S_1, we show the values of α, β, ETT at θ_0 and ETT at θ_1 at the 8 corners of a cube in Figure 7. We notice that the most significant effect is on α due to a change in S_1, the slope of the rejection line. This risk increases roughly two-fold for all combinations of h_1 and h_2. Obviously such a large increase is caused by a significant increase in the size of the rejection zone. Also, note that corresponding to this increase in α, there is a decrease in β and in ETT. Again the reason is the same as for the increase in α.

Next, we study the effect of changing S_1 and S_2, keeping all other factors constant. The results from this case are given in Table 13. Again we see that for constant S_2, S_1 has a very

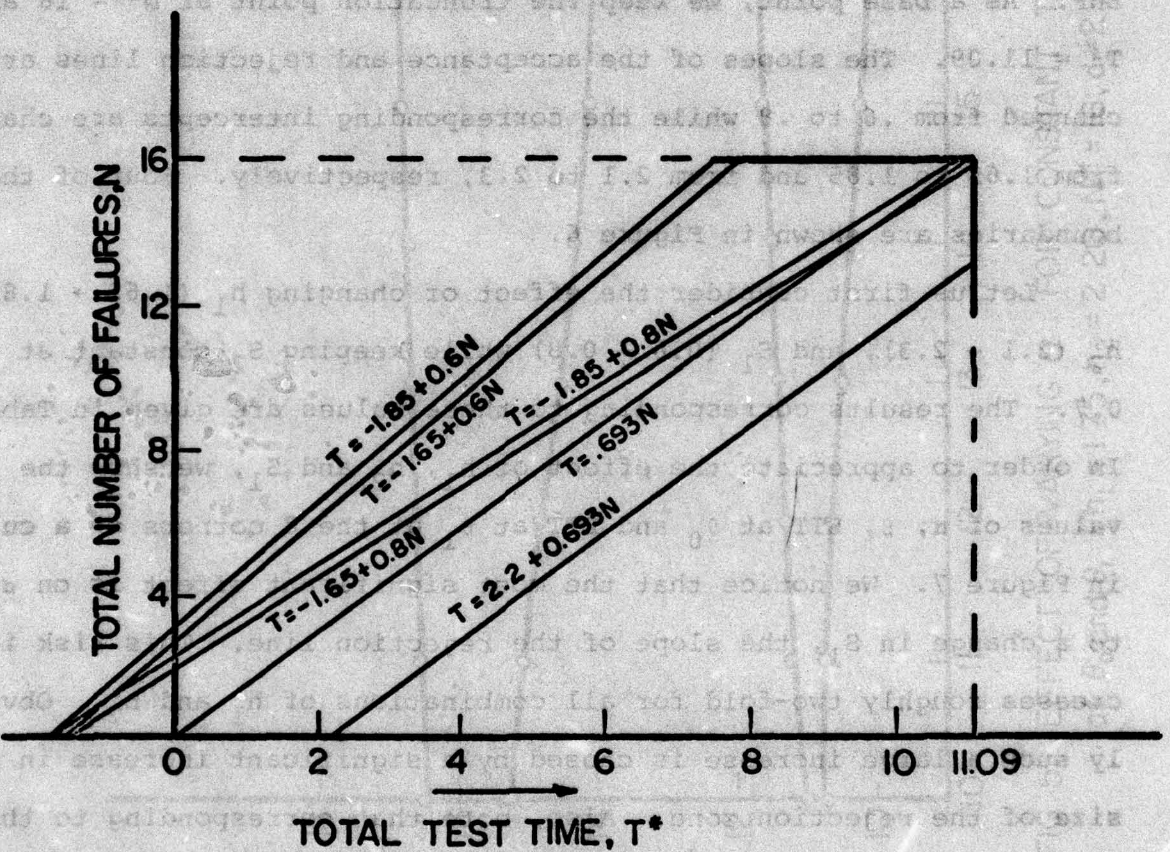

FIG.6 DECISION BOUNDARIES WITH VARYING INTERCEPTS AND SLOPES

TABLE 12

EFFECTS OF CHANGING INTERCEPTS AND SLOPES OF DECISION BOUNDARIES ON RISKS AND ETT (PLAN NO. 3)

($N^* = 16$, $T^* = 11.09$, $\theta_0/\theta_1 = 2$)

Number	h_1	h_2	S_1	S_2	α	β	Ett at θ_0	Ett at $\frac{\theta_0+\theta_1}{2}$	Ett at θ_1
1	1.65	2.1	0.6	0.7	.10	.12	5.43	6.39	4.90
2	1.85	2.1	0.6	0.7	.09	.12	5.53	6.63	5.30
3	1.65	2.3	0.6	0.7	.11	.11	5.88	6.79	5.05
4	1.85	2.3	0.6	0.7	.10	.11	5.98	7.03	5.42
5	1.65	2.1	0.8	0.7	.25	.09	4.17	4.06	2.68
6	1.85	2.1	0.8	0.7	.22	.10	4.41	4.41	2.95
7	1.65	2.3	0.8	0.7	.26	.08	4.56	4.37	2.76
8	1.85	2.3	0.8	0.7	.22	.08	4.81	4.74	3.04

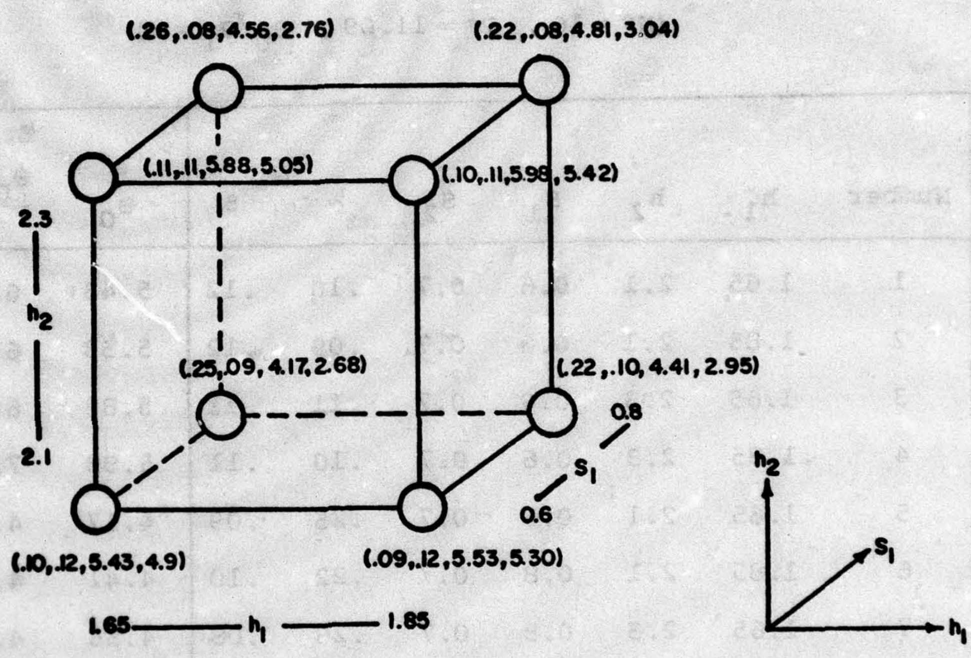

FIG. 7 VALUES OF (α, β, ETT AT θ_0 AND ETT AT θ_1) FOR VARIOUS COMBINATIONS OF h_1, h_2 AND S_1 ($S_2 = 0.7$)

TABLE 13

EFFECTS OF CHANGING SLOPES OF DECISION BOUNDARIES ON RISKS AND ETT (PLAN NO. 3)

(h_1=1.75, h_2 = 2.20, N*=16, T*=11.09; θ_0/θ_1=2)

Number	S_1	S_2	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1
1	.6	.6	.08	.16	4.87	5.82	4.91
2	.7	.6	.13	.15	4.43	4.80	3.58
3	.8	.6	.22	.12	3.88	3.82	2.70
4	.6	.7	.10	.11	5.71	6.72	5.80
5	.7	.7	.15	.10	5.20	5.61	3.82
6	.8	.7	.24	.09	4.49	4.40	2.86
7	.6	.8	.11	.10	6.49	6.38	5.33
8	.7	.8	.15	.09	5.95	6.23	3.96
9	.8	.8	.25	.07	5.15	4.93	2.96

significant effect on α. This is seen to be true for all three values of S_2. It is also interesting to note the tradeoffs that one gets between α, β and ETT as S_1 and S_2 are changed simultaneously while keeping the truncation point unchanged.

In order to study the effects of changing h_1, h_2, S_1 and S_2 for all the plans in MIL-STD-781B (1967), we change these quantities over the appropriate regions and compute the resulting values of α, β, ETT and ENF. These are given in Tables 14 through 23 for plans 1 through 9 (including 4A) of MIL-STD-781B.

TABLE 14

EFFECTS OF CHANGING INTERCEPTS AND SLOPES OF DECISION BOUNDARIES ON RISKS AND ETT (TEST NO. 1)

($N^* = 41$, $T^* = 33.2$, $\theta_0/\theta_1 = 1.5$)

Number	h_1	h_2	S_1	S_2	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1	ENF at θ_0	ENF at $\frac{\theta_0+\theta_1}{2}$	ENF at θ_1
1	4.35	4.35	0.75	0.75	0.085	0.169	15.35	19.46	17.94	15.35	23.36	26.91
2	4.35	4.35	0.85	0.75	0.132	0.153	13.91	15.64	12.63	13.91	18.77	18.95
3	4.35	4.35	0.75	0.85	0.105	0.115	19.48	23.35	19.23	19.48	28.01	28.84
4	4.35	4.35	0.85	0.85	0.151	0.102	17.82	19.23	13.76	17.82	23.07	20.64
5	4.45	4.35	0.75	0.75	0.084	0.169	15.40	19.60	18.20	15.40	23.52	27.31
6	4.45	4.35	0.85	0.75	0.127	0.154	14.02	15.86	12.89	14.02	19.04	19.34
7	4.45	4.35	0.75	0.85	0.104	0.115	19.53	23.49	19.49	19.53	28.19	29.24
8	4.45	4.35	0.85	0.85	0.147	0.103	17.95	19.48	14.03	17.95	23.38	21.05
9	4.35	4.45	0.75	0.75	0.087	0.163	15.68	19.78	18.09	15.68	23.74	27.13
10	4.35	4.45	0.85	0.75	0.133	0.147	14.22	15.93	12.75	14.22	19.11	19.13
11	4.35	4.45	0.75	0.85	0.106	0.112	19.82	23.61	19.31	19.82	28.33	28.97
12	4.35	4.45	0.85	0.85	0.152	0.099	18.16	19.48	13.84	18.16	23.38	20.76
13	4.45	4.45	0.75	0.75	0.086	0.163	15.72	19.92	18.35	15.72	23.91	27.52
14	4.45	4.45	0.85	0.75	0.129	0.148	14.33	16.16	13.01	14.33	19.39	19.52
15	4.45	4.45	0.75	0.85	0.105	0.112	19.87	23.75	19.58	19.87	28.50	29.37
16	4.45	4.45	0.85	0.85	0.148	0.100	18.29	19.74	14.11	18.29	23.68	21.16

TABLE 15

EFFECTS OF CHANGING INTERCEPTS AND SLOPES OF DECISION BOUNDARIES ON RISKS AND ETT (TEST NO. 2)

($N^* = 19$, $T^* = 15.4$, $\theta_0/\theta_1 = 1.5$)

Number	h_1	h_2	S_1	S_2	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1	ENF at θ_0	ENF at $\frac{\theta_0+\theta_1}{2}$	ENF at θ_1
1	2.25	2.75	0.75	0.75	0.197	0.235	7.43	7.95	7.13	7.43	9.54	10.69
2	2.25	2.75	0.85	0.75	0.266	0.233	6.40	6.40	5.40	6.40	7.68	8.10
3	2.25	2.75	0.75	0.85	0.221	0.209	8.61	8.98	7.67	8.61	10.78	11.50
4	2.25	2.75	0.85	0.85	0.288	0.183	7.45	7.31	5.86	7.45	8.77	8.78
5	2.35	2.75	0.75	0.75	0.191	0.255	7.53	8.11	7.34	7.54	9.73	11.01
6	2.35	2.75	0.85	0.75	0.255	0.227	6.54	6.59	5.60	6.54	7.91	8.40
7	2.35	2.75	0.75	0.85	0.215	0.211	8.72	9.16	7.88	8.72	10.99	11.82
8	2.35	2.75	0.85	0.85	0.277	0.186	7.62	7.52	6.07	7.62	9.03	9.10
9	2.25	2.85	0.75	0.75	0.202	0.242	7.67	8.16	7.26	7.67	9.79	10.89
10	2.25	2.85	0.85	0.75	0.271	0.213	6.62	6.59	5.51	6.62	7.91	8.26
11	2.25	2.85	0.75	0.85	0.224	0.202	8.85	9.18	7.77	8.85	11.01	11.65
12	2.25	2.85	0.85	0.85	0.291	0.176	7.68	7.49	5.95	7.68	8.99	8.92
13	2.35	2.85	0.75	0.75	0.196	0.245	7.77	8.33	7.47	7.77	9.99	11.20
14	2.35	2.85	0.85	0.75	0.260	0.218	6.76	6.79	5.71	6.76	8.14	8.56
15	2.35	2.85	0.75	0.85	0.218	0.204	8.96	9.35	7.98	8.96	11.22	11.97
16	2.35	2.85	0.85	0.85	0.281	0.180	7.85	7.71	6.16	7.85	9.25	9.24

TABLE 16

EFFECTS OF CHANGING INTERCEPTS AND SLOPES OF DECISION BOUNDARIES ON RISKS AND ETT (TEST NO. 3)

($N^* = 16$, $T^* = 11.1$, $\theta_0/\theta_1 = 2.0$)

							ETT at			ENF at		
Number	h_1	h_2	S_1	S_2	α	β	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1	θ_0	$\frac{\theta_0+\theta_1}{2}$	θ_1
1	1.50	2.15	0.65	0.65	0.132	0.127	4.76	5.27	3.38	4.76	7.02	7.66
2	1.50	2.15	0.65	0.75	0.217	0.108	4.11	4.10	2.78	4.11	5.46	5.56
3	1.50	2.15	0.65	0.75	0.143	0.099	5.53	6.01	4.02	5.53	8.02	8.05
4	1.50	2.15	0.75	0.75	0.228	0.083	4.77	4.70	2.93	4.77	6.27	5.86
5	1.60	2.15	0.65	0.65	0.122	0.129	4.85	5.45	4.03	4.85	7.26	8.06
6	1.60	2.15	0.75	0.65	0.201	0.112	4.23	4.27	2.93	4.23	5.70	5.86
7	1.60	2.15	0.65	0.75	0.133	0.101	5.64	6.21	4.23	5.64	8.28	8.46
8	1.60	2.15	0.75	0.75	0.212	0.085	4.91	4.91	3.09	4.91	6.54	6.18
9	1.50	2.25	0.65	0.65	0.135	0.118	4.97	5.47	3.90	4.97	7.29	7.79
10	1.50	2.25	0.75	0.65	0.220	0.099	4.30	4.26	2.83	4.30	5.68	5.67
11	1.50	2.25	0.65	0.75	0.145	0.093	5.76	6.19	4.08	5.76	8.26	8.15
12	1.50	2.25	0.75	0.75	0.231	0.076	4.98	4.87	2.97	4.98	6.49	5.95
13	1.60	2.25	0.65	0.65	0.125	0.120	5.07	5.65	4.10	5.07	7.53	8.20
14	1.60	2.25	0.75	0.65	0.204	0.102	4.42	4.45	2.98	4.42	5.93	5.97
15	1.60	2.25	0.65	0.75	0.134	0.094	5.86	6.39	4.28	5.86	8.52	8.56
15	1.60	2.25	0.75	0.75	0.214	0.079	5.12	5.08	3.13	5.12	6.77	6.26

TABLE 17

EFFECTS OF CHANGING INTERCEPTS AND SLOPES OF DECISION BOUNDARIES ON RISKS AND ETT (TEST NO. 4)

($N^* = 8$, $T^* = 5.55$, $\theta_0/\theta_1 = 2.0$)

Number	h_1	h_2	S_1	S_2	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1	EMF at θ_0	EMF at $\frac{\theta_0+\theta_1}{2}$	EMF at θ_1
1	1.00	1.35	0.65	0.65	0.214	0.224	2.37	2.41	1.95	2.37	3.22	3.91
2	1.00	1.35	0.75	0.65	0.282	0.200	2.06	1.99	1.54	2.06	2.65	3.08
3	1.00	1.35	0.65	0.75	0.228	0.197	2.60	2.62	2.05	2.60	3.50	4.09
4	1.00	1.35	0.75	0.75	0.295	0.176	2.25	2.16	1.61	2.25	2.88	3.23
5	1.10	1.35	0.65	0.65	0.197	0.230	2.47	2.55	2.10	2.47	3.40	4.20
6	1.10	1.35	0.75	0.65	0.260	0.207	2.16	2.11	1.65	2.16	2.81	3.30
7	1.10	1.35	0.65	0.75	0.211	0.202	2.70	2.76	2.20	2.70	3.68	4.39
8	1.10	1.35	0.75	0.75	0.273	0.182	2.36	2.29	1.73	2.36	3.06	3.47
9	1.00	1.45	0.65	0.65	0.222	0.204	2.54	2.56	2.03	2.54	3.41	4.06
10	1.00	1.45	0.75	0.65	0.291	0.181	2.21	2.11	1.60	2.21	2.82	3.20
11	1.00	1.45	0.65	0.75	0.235	0.181	2.77	2.76	2.12	2.77	3.68	4.23
12	1.00	1.45	0.75	0.75	0.303	0.160	2.41	2.29	1.67	2.41	3.05	3.34
13	1.10	1.45	0.65	0.65	0.205	0.210	2.64	2.70	2.18	2.64	3.60	4.36
14	1.10	1.45	0.75	0.65	0.269	0.188	2.31	2.24	1.72	2.31	2.99	3.44
15	1.10	1.45	0.65	0.75	0.218	0.186	2.87	2.91	2.27	2.87	3.88	4.53
16	1.10	1.45	0.75	0.75	0.281	0.166	2.52	2.42	1.79	2.52	3.23	3.59

TABLE 18

EFFECTS OF CHANGING INTERCEPTS AND SLOPES OF DECISION BOUNDARIES ON RISKS AND ETT (TEST NO. 4A)

($N^* = 3$, $T^* = 1.65$, $\theta_0/\theta_1 = 3.0$)

Number	h_1	h_2	S_1	S_2	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1	ENP at θ_0	ENP at $\frac{\theta_0+\theta_1}{2}$	ENP at θ_1
1	0.95	0.85	0.50	0.50	0.201	0.173	1.12	1.12	0.87	1.12	1.67	2.62
2	0.95	0.85	0.60	0.50	0.207	0.171	1.11	1.08	0.82	1.11	1.63	2.46
3	0.95	0.85	0.50	0.60	0.206	0.167	1.15	1.14	0.88	1.15	1.70	2.63
4	0.95	0.85	0.60	0.60	0.212	0.165	1.13	1.10	0.82	1.13	1.66	2.47
5	1.05	0.85	0.50	0.50	0.201	0.173	1.13	1.12	0.88	1.13	1.68	2.63
6	1.05	0.85	0.60	0.50	0.203	0.172	1.12	1.10	0.85	1.12	1.66	2.55
7	1.05	0.85	0.50	0.60	0.206	0.167	1.15	1.14	0.88	1.15	1.71	2.64
8	1.05	0.85	0.60	0.60	0.208	0.166	1.14	1.12	0.86	1.14	1.69	2.57
9	0.95	0.95	0.50	0.50	0.212	0.154	1.21	1.18	0.89	1.21	1.78	2.68
10	0.95	0.95	0.60	0.50	0.218	0.152	1.20	1.15	0.84	1.20	1.73	2.52
11	0.95	0.95	0.50	0.60	0.215	0.151	1.24	1.20	0.90	1.24	1.80	2.70
12	0.95	0.95	0.60	0.60	0.221	0.149	1.22	1.17	0.85	1.22	1.76	2.54
13	1.05	0.95	0.50	0.50	0.212	0.154	1.22	1.19	0.90	1.22	1.78	2.69
14	1.05	0.95	0.60	0.50	0.214	0.153	1.21	1.17	0.87	1.21	1.76	2.62
15	1.05	0.95	0.50	0.60	0.215	0.151	1.24	1.20	0.90	1.24	1.81	2.71
16	1.05	0.95	0.60	0.60	0.217	0.150	1.23	1.19	0.88	1.23	1.79	2.63

TABLE 19

EFFECTS OF CHANGING INTERCEPTS AND SLOPES OF DECISION BOUNDARIES ON RISKS AND ETT (TEST NO. 5)

($N^* = 7$, $T^* = 3.85$, $\theta_0/\theta_1 = 3.0$)

Number	h_1	h_2	S_1	S_2	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1	ENF at θ_0	ENF at $\frac{\theta_0+\theta_1}{2}$	ENF at θ_1
1	0.85	1.20	0.50	0.50	0.108	0.102	1.97	2.13	1.43	1.97	3.20	4.28
2	0.85	1.20	0.60	0.50	0.166	0.091	1.78	1.79	1.08	1.78	2.68	3.25
3	0.85	1.20	0.50	0.60	0.114	0.087	2.15	2.30	1.46	2.15	3.46	4.38
4	0.85	1.20	0.60	0.60	0.172	0.077	1.94	1.94	1.11	1.94	2.91	3.33
5	0.95	1.20	0.50	0.50	0.096	0.104	2.01	2.22	1.54	2.01	3.33	4.63
6	0.95	1.20	0.60	0.50	0.144	0.095	1.84	1.90	1.18	1.84	2.85	3.55
7	0.95	1.20	0.50	0.60	0.103	0.089	2.19	2.40	1.58	2.19	3.59	4.73
8	0.95	1.20	0.60	0.60	0.150	0.080	2.01	2.06	1.21	2.01	3.09	3.63
9	0.85	1.30	0.50	0.50	0.112	0.088	2.12	2.27	1.46	2.12	3.40	4.38
10	0.85	1.30	0.60	0.50	0.170	0.078	1.92	1.91	1.11	1.92	2.87	3.33
11	0.85	1.30	0.50	0.60	0.117	0.077	2.29	2.43	1.48	2.29	3.64	4.45
12	0.85	1.30	0.60	0.60	0.175	0.067	2.08	2.06	1.13	2.08	3.09	3.40
13	0.95	1.30	0.50	0.50	0.100	0.090	2.16	2.35	1.58	2.16	3.53	4.73
14	0.95	1.30	0.60	0.50	0.148	0.081	1.99	2.02	1.21	1.99	3.04	3.63
15	0.95	1.30	0.50	0.60	0.106	0.079	2.34	2.52	1.60	2.34	3.78	4.81
16	0.95	1.30	0.60	0.60	0.154	0.070	2.16	2.18	1.23	2.16	3.27	3.70

TABLE 20

EFFECTS OF CHANGING INTERCEPTS AND SLOPES OF DECISION BOUNDARIES ON RISKS AND ETT (TEST NO. 6).

($N^* = 4$, $T^* = 1.61$, $\theta_0/\theta_1 = 5.0$)

Number	h_1	h_2	S_1	S_2	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1	ENF at θ_0	ENF at $\frac{\theta_0+\theta_1}{2}$	ENF at θ_1
1	0.30	0.50	0.35	0.35	0.124	0.130	0.61	0.63	0.38	0.61	1.04	1.92
2	0.30	0.50	0.45	0.35	0.215	0.113	0.52	0.49	0.27	0.52	0.81	1.35
3	0.30	0.50	0.35	0.45	0.130	0.113	0.65	0.68	0.39	0.65	1.13	1.97
4	0.30	0.50	0.45	0.45	0.219	0.102	0.54	0.52	0.27	0.54	0.86	1.37
5	0.40	0.50	0.35	0.35	0.077	0.139	0.66	0.72	0.48	0.66	1.19	2.41
6	0.40	0.50	0.45	0.35	0.146	0.126	0.58	0.58	0.35	0.58	0.97	1.75
7	0.50	0.50	0.35	0.45	0.084	0.119	0.71	0.77	0.49	0.71	1.29	2.47
8	0.40	0.50	0.45	0.45	0.151	0.110	0.62	0.63	0.36	0.62	1.05	0.79
9	0.30	0.60	0.35	0.35	0.133	0.090	0.73	0.74	0.41	0.73	1.24	2.07
10	0.30	0.60	0.45	0.35	0.226	0.077	0.62	0.58	0.29	0.62	0.97	1.45
11	0.30	0.60	0.35	0.45	0.138	0.078	0.78	0,80	0.42	0.78	1.33	2.11
12	0.30	0.60	0.45	0.45	0.230	0.068	0.66	0.62	0.30	0.66	1.03	1.48
13	0.40	0.60	0.35	0.35	0.086	0.097	0.79	0.84	0.51	0.79	1.40	2.57
14	0.40	0.60	0.45	0.35	0.155	0.086	0.70	0.69	0.38	0.70	1.16	1.88
15	0.40	0.60	0.35	0.45	0.092	0.083	0.85	0.90	0.52	0.85	1.50	2.61
16	0.40	0.60	0.45	0.45	0.160	0.075	0.75	0.74	0.38	0.75	1.24	2.91

TABLE 21

EFFECTS OF CHANGING INTERCEPTS AND SLOPES OF DECISION BOUNDARIES ON RISKS AND ETT (TEST NO. 7)

($N^* = 6$, $T^* = 4.87$, $\theta_0/\theta_1 = 1.5$)

Number	h_1	h_2	s_1	s_2	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1	ENF at θ_0	ENF at $\frac{\theta_0+\theta_1}{2}$	ENF at θ_1
1	1.55	2.05	0.75	0.75	0.351	0.285	3.43	3.33	3.00	3.43	3.99	4.50
2	1.55	2.05	0.85	0.75	0.372	0.273	3.24	3.09	2.72	3.24	3.71	4.08
3	1.55	2.05	0.75	0.85	0.357	0.278	3.53	3.40	3.05	3.53	4.08	4.57
4	1.55	2.05	0.85	0.85	0.377	0.266	3.34	3.16	2.77	3.34	3.80	4.15
5	1.65	2.05	0.75	0.75	0.348	0.287	3.47	3.38	3.06	3.47	4.05	4.60
6	1.65	2.05	0.85	0.75	0.364	0.277	3.31	3.17	2.81	3.31	3.80	4.22
7	1.65	2.05	0.75	0.85	0.353	0.280	3.57	3.45	3.11	3.57	4.14	4.66
8	1.65	2.05	0.85	0.85	0.369	0.270	3.40	3.24	2.86	3.40	3.89	4.28
9	1.55	2.15	0.75	0.75	0.356	0.278	3.52	3.40	3.05	3.52	4.08	4.57
10	1.55	2.15	0.85	0.75	0.377	0.266	3.33	3.16	2.77	3.33	3.79	4.15
11	1.55	2.15	0.75	0.85	0.360	0.273	3.62	3.47	3.08	3.62	4.16	4.63
12	1.55	2.15	0.85	0.85	0.381	0.261	3.42	3.23	2.81	3.42	3.87	4.21
13	1.65	2.15	0.75	0.75	0.353	0.280	3.56	3.45	3.11	3.56	4.14	4.66
14	1.65	2.15	0.85	0.75	0.369	0.270	3.40	3.24	2.86	3.40	3.89	4.28
15	1.65	2.15	0.75	0.85	0.357	0.275	3.66	3.52	3.15	3.66	4.22	4.72
16	1.65	2.15	0.85	0.85	0.373	0.265	3.49	3.31	2.89	3.49	3.97	4.34

TABLE 22

EFFECTS OF CHANGING INTERCEPTS AND SLOPES OF DECISION BOUNDARIES ON RISKS AND ETT (TEST NO. 8)

($N^* = 3$, $T^* = 2.08$, $\theta_0/\theta_1 = 2.0$)

Number	h_1	h_2	S_1	S_2	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1	ENF at θ_0	ENF at $\frac{\theta_0+\theta_1}{2}$	ENF at θ_1
1	6.00	0.80	0.65	0.65	0.258	0.342	1.17	1.17	1.08	1.17	1.57	2.17
2	6.00	0.80	0.75	0.65	0.258	0.342	1.17	1.17	1.08	1.17	1.57	2.17
3	6.00	0.80	0.65	0.75	0.266	0.331	1.20	1.20	1.10	1.20	1.60	2.19
4	6.00	0.80	0.75	0.75	0.266	0.331	1.20	1.20	1.10	1.20	1.60	2.19
5	6.10	0.80	0.65	0.65	0.258	0.342	1.17	1.17	1.08	1.17	1.57	2.17
6	6.10	0.80	0.75	0.65	0.258	0.342	1.17	1.17	1.08	1.17	1.57	2.17
7	6.10	0.80	0.65	0.75	0.266	0.331	1.20	1.20	1.10	1.20	1.60	2.19
8	6.10	0.80	0.75	0.75	0.266	0.331	1.20	1.20	1.10	1.20	1.60	2.19
9	6.00	0.90	0.65	0.65	0.279	0.308	1.27	1.26	1.14	1.27	1.68	2.28
10	6.00	0.90	0.75	0.65	0.279	0.308	1.27	1.26	1.14	1.27	1.68	2.28
11	6.00	0.90	0.65	0.75	0.286	0.299	1.30	1.28	1.15	1.30	1.71	2.30
12	6.00	0.90	0.75	0.75	0.286	0.299	1.30	1.28	1.15	1.30	1.71	2.30
13	6.10	0.90	0.65	0.65	0.279	0.308	1.27	1.26	1.14	1.27	1.68	2.28
14	6.10	0.90	0.75	0.65	0.279	0.308	1.27	1.26	1.14	1.27	1.68	2.28
15	6.10	0.90	0.65	0.75	0.286	0.299	1.30	1.28	1.15	1.30	1.71	2.30
16	6.10	0.90	0.75	0.75	0.286	0.299	1.30	1.28	1.15	1.30	1.71	2.30

TABLE 23

EFFECTS OF CHANGING INTERCEPTS AND SLOPES OF DECISION BOUNDARIES ON RISKS AND ETT (TEST NO. 9)

($N^* = 9$, $T^* = 8.03$, $\theta_0/\theta_1 = 1.25$)

Number	h_1	h_2	S_1	S_2	α	β	ETT at θ_0	ETT at $\frac{\theta_0+\theta_1}{2}$	ETT at θ_1	ENF at θ_0	ENF at $\frac{\theta_0+\theta_1}{2}$	ENF at θ_1
1	3.10	1.95	0.85	0.85	0.330	0.433	4.84	4.95	4.95	4.84	5.49	6.18
2	3.10	1.95	0.95	0.85	0.336	0.428	4.65	4.71	4.66	4.65	5.24	5.82
3	3.10	1.95	0.85	0.95	0.352	0.406	5.13	5.21	5.16	5.13	5.79	6.45
4	3.10	1.95	0.95	0.95	0.357	0.402	4.95	4.98	4.87	4.95	5.53	6.09
5	3.20	1.95	0.85	0.85	0.330	0.433	4.85	4.97	4.98	4.85	5.52	6.22
6	3.20	1.95	0.95	0.85	0.334	0.429	4.69	4.76	4.71	4.69	5.29	5.89
7	3.20	1.95	0.85	0.95	0.351	0.406	5.15	5.23	5.19	5.15	5.82	6.49
8	3.20	1.95	0.95	0.95	0.356	0.403	4.99	5.02	4.93	4.99	5.58	6.16
9	3.10	2.05	0.85	0.85	0.341	0.419	5.00	5.10	5.08	5.00	5.66	6.35
10	3.10	2.05	0.95	0.85	0.346	0.414	4.82	4.87	4.78	4.82	5.41	5.98
11	3.10	2.05	0.85	0.95	0.360	0.395	5.29	5.35	5.28	5.29	5.94	6.60
12	3.10	2.05	0.95	0.95	0.366	0.391	5.11	5.12	4.99	5.11	5.69	6.23
13	3.20	2.05	0.85	0.85	0.340	0.419	5.02	5.12	5.11	5.02	5.69	6.39
14	3.20	2.05	0.95	0.85	0.345	0.415	4.86	4.91	4.84	4.86	5.46	6.05
15	3.20	2.05	0.85	0.95	0.360	0.395	5.31	5.37	5.31	5.31	5.97	6.64
16	3.20	2.05	0.95	0.95	0.364	0.392	5.15	5.17	5.04	5.15	5.74	6.30

7. CONCLUDING REMARKS

In this report we have briefly described the truncated sequential probability ratio test for reliability testing in the exponential case. Using a newly developed exact method for the computations of OC and ETT functions, we have studied how, and to what extent, the risks, ETT and ENF are affected by changing the truncation regions and the decision boundaries.

This investigation has provided a useful insight into the behavior of quantities of interest (α, β, ETT, ENF) as the test region is systematically varied. The study also presented a framework for exploring alternative decision regions to conduct tradeoff studies between α, β, ETT and ENF.

8. REFERENCES

[1] Anderson, T.W. (1960), "A modification of the sequential probability ratio test to reduce the sample size," Annals of Mathematical Statistics, vol. 31, pp. 165-197.

[2] Armitage, P. (1957), "Restricted sequential procedures," Biometrika 44, pp. 1-15.

[3] Aroian, L.A. (1963), "Exact truncated sequential tests for the exponential density function," Proceedings of the 9th International Symposium on Reliability and Quality Control, pp. 470-486.

[4] Aroian, L.A. (1976), "Application of the direct method in sequential analysis," Technometrics, vol. 18, No. 3, pp. 301-306.

[5] Epstein, B. and Sobel, M. (1955), "Sequential life tests in the exponential case," Annals of Mathematical Statistics, vol. 26, pp. 82-93.

[6] Epstein, B., Patterson, A.A., and Qualls, C.R. (1963), "The exact analysis of sequential life tests with particular application to AGREE Plans," Proceedings Aerospace Reliability and Maintainability Conference, pp. 284-311.

[7] Ghosh, B.K. (1970), "Sequential tests of statistical hypotheses," Addison-Wesley

[8] MIL-STD-781B (1967), "Reliability Tests: Exponential Distributions," Department of Defense, November 15, 1967.

[9] Raghavachari, M. (1965), "Operating characteristic and expected sample size of SPRT for the simple exponential distribution," Calcutta Statistical Association Bulletin, vol. 14, pp. 65-73.

[10] Wald, A. (1947), "Sequential Analysis," John Wiley, New York,

[11] Weiss, L. (1953), "Testing one simple hypothesis against another," Annals of Mathematical Statistics, vol. 24, pp. 273-281.

APPENDIX A

EXPRESSIONS FOR EXACT RISKS AND ETT

In this Appendix we give expressions for the exact computations of the producer's risk (α'), the consumer's risk (β') and the expected test time (ETT) for reliability testing using a truncated SPRT.

Let $T_1, T_2, \ldots$ be independent, identically distributed exponential random variables with parameter θ. We want to test the hypothesis

$$H_0: \ \theta = \theta_0$$
$$\text{vs. } H_1: \ \theta = \theta_1 \qquad (\theta_0 > \theta_1)$$

by using a truncated SPRT.

Let $f_N(w|\theta)$ be the pdf of $W_N = \sum_{i=1}^{N} t_i$ provided that we did not stop testing before the Nth sample and that the parameter value is θ. Then it can be shown that

$$f_N(w|\theta) = [\sum_{i=e(N-1)}^{N} {}_N I_i(w) \cdot {}_N g_i(w)] \exp(-w/\theta) \cdot \theta^{-N} \qquad \text{(A-1)}$$

where $e(N-1)$, ${}_N I_i(w)$ and ${}_N g_i(w)$ are defined as follows.

(i) $e(N-1) = \min\{i: \ A_i > R_{N-1}\}$

(ii)
$${}_N I_i(w) = \begin{cases} I_{[R_{N-1}, A_{e(N-1)})}(w) & \text{for } i = e(N-1) \\ I_{[A_{i-1}, A_i)}(w) & \text{for } e(N-1) < i < N \\ I_{[A_{N-1}, \infty)}(w) & \text{for } i = N \end{cases}$$

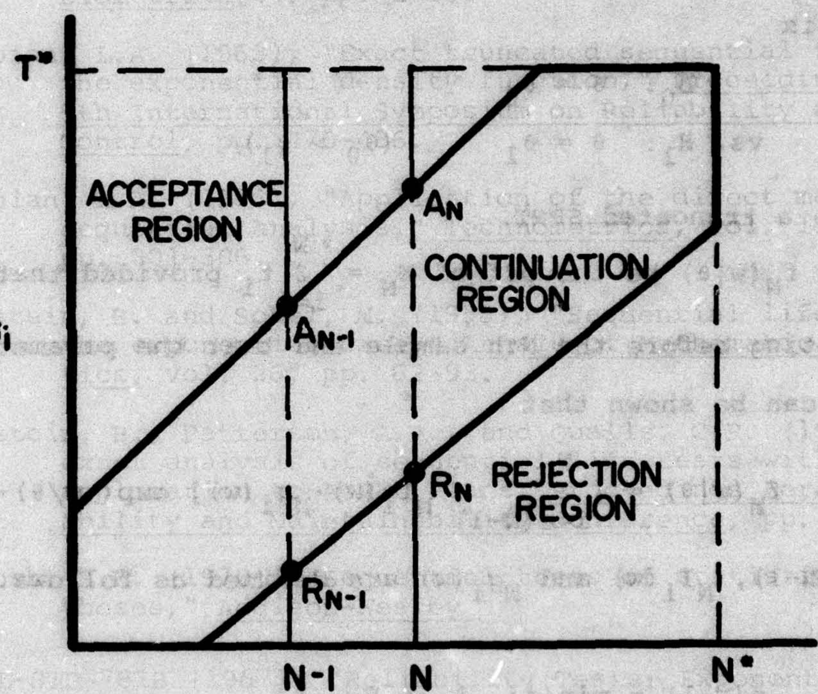

FIG. A-1 BOUNDARIES FOR A TRUNCATED SPRT

(iii) $${}_N g_i(w) = \sum_{j=1}^{N-i+1} {}^N h_{i,j}\, w^{j-1},$$

where ${}^N h_{i,j}$ is given by the following expressions.

<u>N=2</u>

$${}^2h_{1,1} = -R_1,\quad {}^2h_{i,2} = 1,\quad {}^2h_{2,2} = (A_1 - R_1),$$

<u>N>2</u>

(a) <u>$i = e(N-1),\quad j=1$</u>

$${}^N h_{e(N-1),1} = \sum_{j=1}^{N-e(N-1)} {}^{N-1}h_{e(n-1),j}\, \frac{(R_{N-1})^j}{j},$$

(b) <u>$e(N-1) < i \le N-1,\quad j=1$</u>

$${}^N h_{i,1} = \int_{R_{N-1}}^{A_{e(N-1)}} {}_{N-1}g_{e(N-1)}(x)\, dx$$

$$+ \sum_{j=e(N-1)+1}^{i-1} \int_{A_{j-1}}^{A_j} {}_{N-1}g_j(x)\, dx$$

$$+ \sum_{j=1}^{(N-1)+i+1} {}^{N-1}h_{i,j}\, \frac{(A_{i-1})^j}{j},$$

(c) <u>$i = N,\quad j=1$</u>

$${}^N h_{N,1} = \int_{R_{N-1}}^{A_{e(N-1)}} {}_{N-1}g_{e(N-1)}(x)\, dx + \sum_{j=e(N-1)+1}^{N-1} \int_{A_{j-1}}^{A_j} {}_{N-1}g_j(x)\, dx,$$

(d) <u>$e(N-1) \le i \le N-1$</u> , <u>$2 \le j \le N-i+2$</u>

$${}^N h_{i,j} = \frac{{}^{N-1}h_{i,(j-1)}}{j-1}$$

It can be shown that the probability of accepting H_o when the value of the parameter is θ is

$$p_N(H_o|\theta) = (\frac{1}{\theta})^{N-1} \, {}^N h_{N,1} \exp(-A_N/\theta).$$

Now, the expressions for α', β' and ETT can be obtained as

$$\left.\begin{aligned} \alpha' &= 1 - \sum_{N=1}^{N^*} p_N(H_o|\theta_o) \\ \beta' &= \sum_{N=1}^{N^*} p_N(H_o|\theta_1) \end{aligned}\right\} \qquad \text{(A-2)}$$

$$ETT(\theta) = \sum_{N=1}^{N^*} [A_N p_n(H_o|\theta) + \int_{R_{N-1}}^{R_N} w \, f_N(w|\theta) \cdot dw \qquad \text{(A-3)}$$

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